

# Spin current assisted magnetization dynamics in exchange coupled magnetic layers

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*Modern Trends in Physics Research, Poznań, 11th February 2015*

# Nanoscale spin torque devices for spin electronics

Joint research project under the framework of Polish-Swiss research programme



[nanospin.agh.edu.pl](http://nanospin.agh.edu.pl)

## Partners

- AGH University of Science and Technology in Kraków  
T. Stobiecki – coordinator
- Institute of Molecular Physics in Poznań, Polish Academy of Sciences  
J. Dubowik – experiment, J. Barnaś – theory
- École Polytechnique Fédérale in Lausanne  
J.-Ph. Ansermet

## Objectives

jointly developing novel **nanoscale spintronic devices** based on the **spin transfer torque effect**, which promises unrivaled future scaling, flexibility and low power consumption



## 1 Introduction

## 2 Current-induced destabilization of a composite free layer

Model

Results

## 3 Current-induced spin wave excitations in YIG double layer

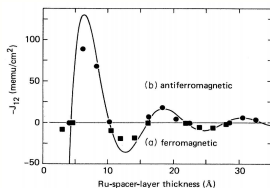
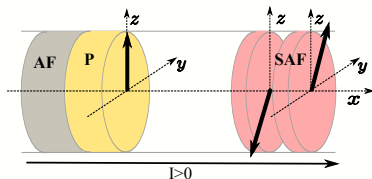
Motivation

Model

Results

## 4 Summary

# Why double layers?



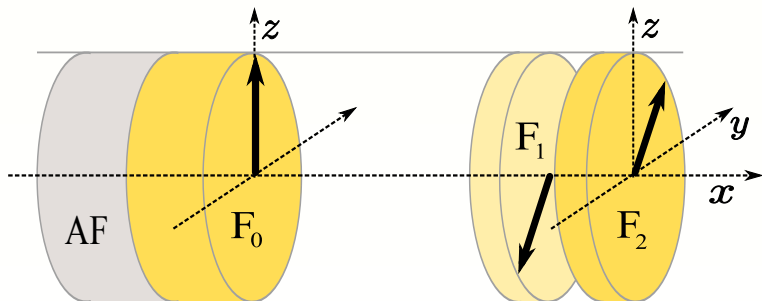
S.S.P. Parkin, D. Mauri

*Spin engineering: Direct determination of the Ruderman-Kittel-Kasuya-Yosida far-field range function in ruthenium*

*Phys. Rev. B* **44**, 7131 (1991)

- Double layer has **high stability**,  $\Delta = \frac{E}{k_B T} > 40$
- **Reduction** of critical current

## Spin valve with composite free layer (CFL)



# Magnetization dynamics

## Landau-Lifshitz-Gilbert equation

$$\frac{d\hat{S}_i}{dt} + \alpha \hat{S}_i \times \frac{d\hat{S}_i}{dt} = \Gamma_i, \quad \Gamma_i = -|\gamma_g| \mu_0 \hat{S}_i \times \mathbf{H}_{\text{eff}i} + \frac{|\gamma_g|}{M_s d_i} \boldsymbol{\tau}_i$$

## Effective magnetic field

$$\mathbf{H}_{\text{eff}i} = -H_{\text{app}} \hat{\mathbf{e}}_z - H_{\text{ani}} (\hat{S}_i \cdot \hat{\mathbf{e}}_z) \hat{\mathbf{e}}_z + \mathbf{H}_{\text{demi}}(\hat{S}_i) + H_{\text{RKKY}i} \hat{S}_j$$

Interlayer exchange coupling  $H_{\text{RKKY}i} = -J_{\text{RKKY}} / (\mu_0 M_s d_i)$

## Spin transfer torque

$$\boldsymbol{\tau}_{1\parallel} = I \hat{S}_1 \times \left[ \hat{S}_1 \times \left( a_1^{(0)} \hat{S}_0 + a_1^{(2)} \hat{S}_2 \right) \right]$$

$$\boldsymbol{\tau}_{1\perp} = I \hat{S}_1 \times \left( b_1^{(0)} \hat{S}_0 + b_1^{(2)} \hat{S}_2 \right)$$

$$\boldsymbol{\tau}_{2\parallel} = I a_2^{(1)} \hat{S}_2 \times (\hat{S}_2 \times \hat{S}_1)$$

$$\boldsymbol{\tau}_{2\perp} = I b_2^{(1)} \hat{S}_2 \times \hat{S}_1$$

# Linearized Landau-Lifshitz-Gilbert equation

## Static points of the dynamics

AP configuration  $\begin{matrix} \uparrow & \downarrow \uparrow \end{matrix}$   $\hat{S}_1 = (0, 0, -1)$ ,  $\hat{S}_2 = (0, 0, 1)$

P configuration  $\begin{matrix} \uparrow & \uparrow \downarrow \end{matrix}$   $\hat{S}_1 = (0, 0, 1)$ ,  $\hat{S}_2 = (0, 0, -1)$

## Linearized LLG

$$\frac{d}{dt} \delta \tilde{S} = \bar{D} \cdot \delta \tilde{S}, \text{ where } \bar{D} = \begin{pmatrix} D_1 & C_{12} \\ C_{21} & D_2 \end{pmatrix}$$

single spin dynamic matrix:

$$D_i = \begin{pmatrix} \omega_i^\tau - \alpha \omega_i^{h1} & \alpha \omega_i^\tau - \omega_i^{h2} \\ -\alpha \omega_i^\tau - \omega_i^{h1} & \omega_i^\tau + \alpha \omega_i^{h2} \end{pmatrix}$$

coupling between the free layer's:

$$C_{ij} = \begin{pmatrix} -\omega_{ij}^\tau + \alpha \omega_{Ji} & -\left( \alpha \omega_{ij}^\tau + \omega_{Ji} \right) \cos \Delta \phi \\ \alpha \omega_{ij}^\tau + \omega_{Ji} & -\left( \omega_{ij}^\tau - \alpha \omega_{Ji} \right) \cos \Delta \phi \end{pmatrix}$$

# Stability analysis

## Lyapunov criterion

Static state,  $\tilde{S}_0$ , is stable if and only if *all the eigenvalues of  $\bar{D}$  have negative real parts*. If one of them becomes positive, the static point is unstable.

**Drawback:** We do not have general expressions for the eigenvalues.

$$\begin{aligned} P(\lambda) &= \det(\bar{D} - \lambda \bar{I}) \\ &= \lambda^4 + c_3 \lambda^3 + c_2 \lambda^2 + c_1 \lambda + c_0 \end{aligned}$$

$$H_P = \begin{pmatrix} c_1 & c_0 & 0 & 0 \\ c_3 & c_2 & c_1 & c_0 \\ 0 & 1 & c_3 & c_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## Routh-Hurwitz theorem

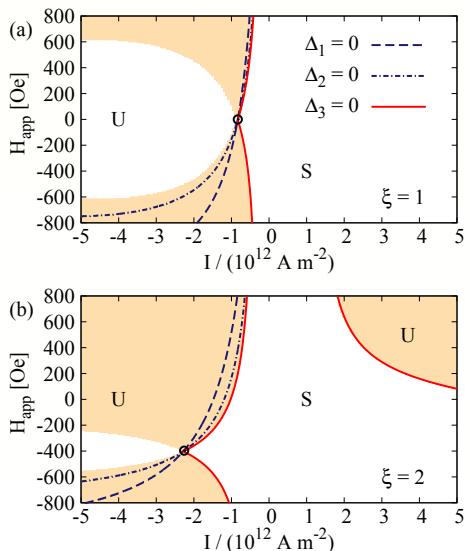
The roots of polynomial  $P(\lambda)$  have all negative real parts if and only if *all the leading principal minors of  $H_P$  are positive*.

$$\Delta_1 = c_1 > 0, \quad \Delta_2 = \det \begin{pmatrix} c_1 & c_0 \\ c_3 & c_2 \end{pmatrix} > 0, \quad \Delta_3 = \det \begin{pmatrix} c_1 & c_0 & 0 \\ c_3 & c_2 & c_1 \\ 0 & 1 & c_3 \end{pmatrix} > 0$$



# Results

## Hurwitz determinants for AP configuration



(a) Synthetic antiferromagnet  
( $\xi = d_1/d_2 = 1$ )

(b) Synthetic ferrimagnet  
( $\xi = 2$ )

- in **S** area all the HDs are positive
- $\Delta_3$  is the first HD which becomes negative
- lines  $\Delta_i$  for  $i = 1, 2, 3$  intersects in the same point



P. Baláz and J. Barnaš

Physical Review B 88, 014406 (2013)

# Hurwitz determinants

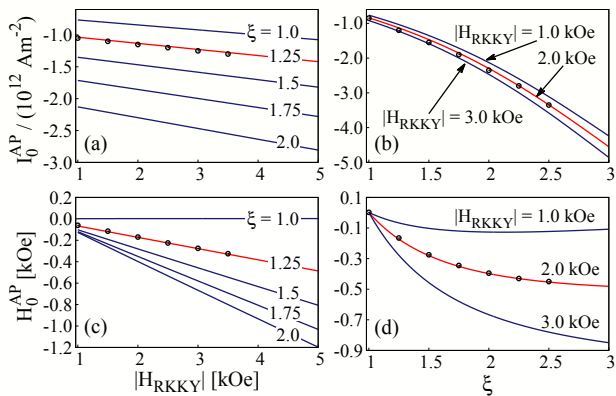
## We observe that

- The **critical current** can be obtained from the condition  $\Delta_3 = 0$ .
- All three Hurwitz determinants **intersects in the same point(s)**.  
When  $\Delta_1 = c_1 = 0$  then  $\Delta_2 = c_0 c_3$  and  $\Delta_3 = c_0 c_3^2$ .  
Intersection points:
  - $c_1 = 0$  and  $c_3 = 0$
  - $c_1 = 0$  and  $c_0 = 0$
- Existence of **double point** in (i).  
 $\partial\Delta_3/\partial I = \partial\Delta_3/\partial H_{\text{app}} = 0$  only in the intersection point (i)
- The **maximum critical current** is given by  $c_3 = \text{Tr}\bar{D} = 0$  and **correspondent field** can be obtained from  $c_1 = 0$ .

## Critical current density in AP configuration

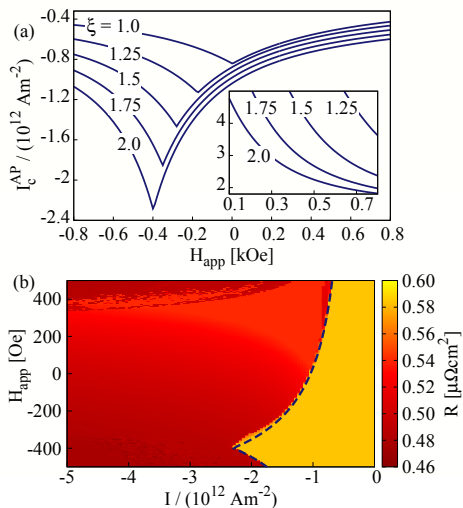
$$I_0^{\text{AP}} = -\alpha \frac{\mu_0 M_s \xi d}{a_1^{(0)} + a_1^{(2)} + \xi a_2^{(1)}} \left[ 2H_{\text{ani}} + H_1^{\text{d}} + H_2^{\text{d}} - (1 + \xi^{-1}) H_{\text{RKKY}} \right]$$

## Results



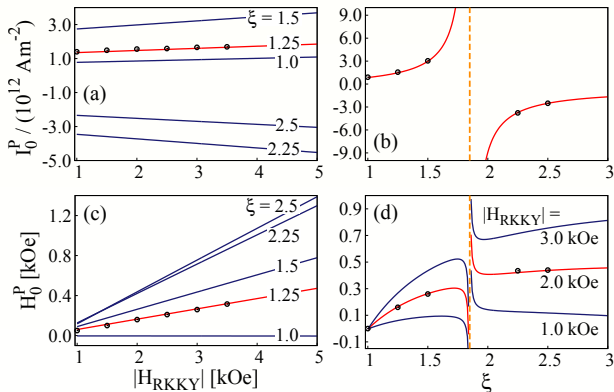
# Results

Critical current density and comparison with numerical simulations



# Results

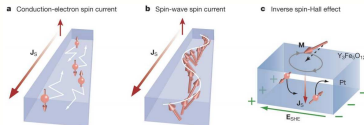
Critical current density in the opposite configuration



$$\text{Critical asymmetry } a_1^{(0)}(\xi_c) - a_1^{(2)}(\xi_c) - \xi_c a_2^{(1)}(\xi_c) = 0$$

In the studied device  $\xi_c \simeq 1.85$ .

# Motivation



A. Kapelrud and A. Brataas.

*Spin Pumping and Enhanced Gilbert Damping in Thin Magnetic Insulator Films*

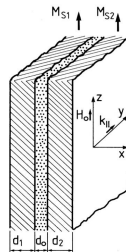
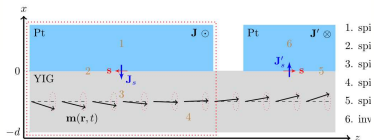
*Physical Review Letters* **111**, 097602 (2013)



Y. Kajiwara *et al.*

*Transmission of electrical signals by spin-wave interconversion in a magnetic insulator*

*Nature* **464**, 262 (2010)



M. Vohl, J. Barnaś, and P. Grünberg

*Effect of interlayer exchange coupling on spin-wave spectra in magnetic double layers: Theory and experiment*

*Physical Review B* **39**, 12003 (1989)

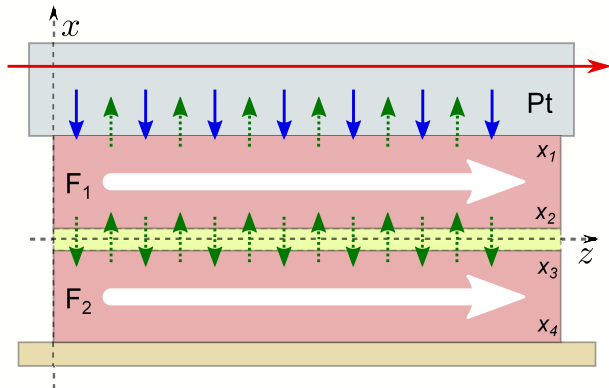


Y. Zhou *et al.*

*Current-induced spin-wave excitation in Pt/YIG bilayer*

*Physical Review B* **88**, 184403 (2013)

## YIG double layer



# Bulk dynamics

## Landau-Lifshitz-Gilbert equation

$$\frac{d\mathbf{M}_j}{dt} = -\gamma_j \mu_0 \mathbf{M}_j \times \mathbf{H}_{\text{eff}j} + \frac{\alpha_j}{M_{sj}} \mathbf{M}_j \times \frac{d\mathbf{M}_j}{dt}$$

## Effective magnetic field in $j$ -th layer

$$\mathbf{H}_{\text{eff}j}(\mathbf{r}, t) = s_0 H_0 \hat{e}_z + \frac{H_{aj}}{M_{sj}} [\mathbf{M}_j(\mathbf{r}, t) \cdot \hat{e}_z] \hat{e}_z + \mathbf{h}_j(\mathbf{r}, t) + \frac{2A_j}{\mu_0 M_{sj}^2} \nabla^2 \mathbf{M}_j(\mathbf{r}, t)$$

approximation:  $\mathbf{M}_j(\mathbf{r}, t) = s_j M_{sj} \hat{e}_z + \mathbf{m}_j(\mathbf{r}, t)$ , where  $\mathbf{m}_j(\mathbf{r}, t) = (m_{j,x}(\mathbf{r}, t), m_{j,y}(\mathbf{r}, t), 0)$

## Dynamic variables

$$\mathbf{m}_j(\mathbf{r}, t) = \mathbf{m}_j(\mathbf{r}) e^{-i\omega t}$$

$$\mathbf{h}_j(\mathbf{r}, t) = \mathbf{h}_j(\mathbf{r}) e^{-i\omega t}$$

## Maxwell equations

$$\nabla \times \mathbf{h}_j(\mathbf{r}) = 0$$

$$\nabla \cdot [\mathbf{h}_j(\mathbf{r}) + \mathbf{m}_j(\mathbf{r})] = 0$$



# Bulk dynamics

Magnetostatic scalar potential:

$$\mathbf{h}_j(\mathbf{r}) = -\nabla\psi_j(\mathbf{r}), \text{ which obeys } \nabla^2\psi_j - \left( \frac{\partial m_{j,x}}{\partial x} + \frac{\partial m_{j,y}}{\partial y} \right) = 0$$

**three independent variables**

$$m_{j,x}(\mathbf{r}) = m_{j,x}(x) e^{i\mathbf{q}\cdot\mathbf{s}}$$

$$m_{j,y}(\mathbf{r}) = m_{j,y}(x) e^{i\mathbf{q}\cdot\mathbf{s}}$$

$$\psi_j(\mathbf{r}) = \psi_j(x) e^{i\mathbf{q}\cdot\mathbf{s}}$$

where  $\mathbf{q} = (q_y, q_z)$ , and  $\mathbf{s} = (y, z)$

# Bulk dynamics

## Solutions

$$m_{j,x} = \sum_{l=1}^3 \left[ C_{j,1}^{(l)} \cos(k_{j,l}x) + D_{j,1}^{(l)} \sin(k_{j,l}x) \right]$$

$$m_{j,y} = \sum_{l=1}^3 \left[ C_{j,2}^{(l)} \cos(k_{j,l}x) + D_{j,2}^{(l)} \sin(k_{j,l}x) \right]$$

$$\psi_j = \sum_{l=1}^3 \left[ C_{j,3}^{(l)} \cos(k_{j,l}x) + D_{j,3}^{(l)} \sin(k_{j,l}x) \right]$$

## wave vectors

$$k_{j,1}^2 = -q_y^2$$

$$k_{j,2(3)}^2 = -\frac{1}{Q_j} \left[ 1/2 + \frac{H_{aj}}{M_{sj}} + \sigma_j \eta_j - i \alpha_j f_j \pm \sqrt{f_j^2 + (1/2)^2} \right] - q_y^2$$

where  $f_j = \omega / \omega_{M,j}$ , and  $\eta_j = H_0 / M_{s,j}$ .

# Bulk dynamics

we reduce number of variables using

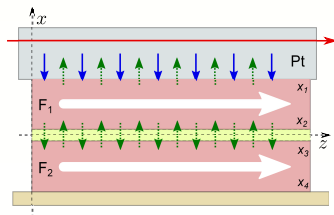
$$\begin{aligned}C_{j,n}^{(l)} &= p_{j,n}^{(l)} C_{j,1}^{(l)} + q_{j,n}^{(l)} D_{j,1}^{(l)} \\D_{j,n}^{(l)} &= -q_{j,n}^{(l)} C_{j,1}^{(l)} + p_{j,n}^{(l)} D_{j,1}^{(l)}\end{aligned}$$

12 independent variables

$C_{j,1}^{(l)}$  and  $D_{j,1}^{(l)}$  for  $l = 1, 2, 3$  and  $j = 1, 2$ .

# Boundary conditions

4 interfaces



$$x_{i1} = L_1 + L_s/2$$

$$x_{i2} = L_s/2$$

$$x_{i3} = -L_s/2$$

$$x_{i4} = -(L_2 + L_s/2)$$

# External interfaces

## Landau-Lifshitz-Gilbert equation

$$\frac{d\mathbf{M}_1}{dt} = -\gamma_1 \mu_0 \mathbf{M}_1 \times \mathbf{H}_{\text{eff}i1} + \frac{\alpha_1}{M_{s1}} \mathbf{M}_1 \times \frac{d\mathbf{M}_1}{dt} + \frac{\gamma_1}{M_{s1}} (\boldsymbol{\tau}_{\text{STT}i1} + \boldsymbol{\tau}_{\text{SP}i1})$$

## Effective magnetic field

$$\mathbf{H}_{\text{eff}i1} = H_0 \hat{e}_z + \mathbf{h}_1(\mathbf{r}, t) + \frac{2A_1}{\mu_0 M_{s1}^2} \nabla^2 \mathbf{M}_1 - \frac{2K_s^1}{\mu_0 M_{s1}^2} (\mathbf{M}_1 \times \hat{n}_1) \hat{n}_1$$

## Spin transfer torque

$$\boldsymbol{\tau}_{\text{STT}i1} = \frac{J_s}{M_{s1} \delta} \mathbf{M}_1 \times \hat{e}_z \times \mathbf{M}_1$$

## Spin pumping

$$\boldsymbol{\tau}_{\text{SP}i1} = \frac{g_{r1} \hbar}{4\pi \delta} \frac{\mathbf{M}_1}{M_{s1}} \times \frac{d\mathbf{M}_1}{dt}$$

# External interfaces

## Rado-Weertman boundary conditions

### Top interface ( $x = x_{i1}$ )

$$\left( A_1 \frac{\partial}{\partial x} - i G_1 \omega \right) m_{1,y} - \frac{J_s}{2} m_{1,x} \Big|_{x_{i1}} = 0$$

$$\left( A_1 \frac{\partial}{\partial x} - i G_1 \omega - K^s_1 \right) m_{1,x} + \frac{J_s}{2} m_{1,y} \Big|_{x_{i1}} = 0.$$

where  $G_1 = \hbar g_{r1}^{(e)} / (8\pi)$  with  $g_{r1}^{(e)}$  being **mixing conductance** of the top interface

### Bottom interface ( $x = x_{i4}$ )

$$\frac{\partial}{\partial x} m_{2,y} \Big|_{x_{i4}} = 0$$

$$\frac{\partial}{\partial x} m_{2,x} \Big|_{x_{i4}} = 0$$

where  $G_2 = \hbar g_{r2}^{(e)} / (8\pi)$  with  $g_{r2}^{(e)}$  being **mixing conductance** of the bottom interface

# Internal interfaces

## Landau-Lifshitz-Gilbert equation

$$\frac{d\mathbf{M}_j}{dt} = -\gamma_j \mu_0 \mathbf{M}_j \times \mathbf{H}_{\text{eff}j}' + \frac{\alpha_j}{M_{sj}} \mathbf{M}_j \times \frac{d\mathbf{M}_j}{dt} + \frac{\gamma_j}{M_{sj}} \boldsymbol{\tau}_{\text{DC}j}$$

## Effective magnetic field

$$\mathbf{H}_{\text{eff}j}' = H_0 \hat{e}_z + \mathbf{h}_j(\mathbf{r}, t) + \frac{2A_j}{\mu_0 M_{sj}^2} \nabla^2 \mathbf{M}_j - \frac{2A_{12}}{\mu_0 M_{sj} M_{si} \delta} \mathbf{M}_i$$

## Dynamic coupling

$$\boldsymbol{\tau}_{\text{DC}j} = \frac{\hbar g_r}{4\pi\delta} \left( \frac{\mathbf{M}_j}{M_{sj}} \times \frac{d\mathbf{M}_j}{dt} - \frac{\mathbf{M}_i}{M_{si}} \times \frac{d\mathbf{M}_i}{dt} \right)$$

## Mixing conductance

$$\frac{1}{g_r} = \frac{1}{g_{r12}} + \frac{1}{g_{r13}}$$

# Internal interfaces

## Hoffmann conditions

### First couple

$$\left( \sigma \frac{A_1}{M_{s1}} \frac{\partial}{\partial x} - \frac{A_{12}}{M_{s1}} + i \sigma \frac{G_{12}}{M_{s1}} \omega \right) m_{1,y} |_{x_{i2}} + \left( \sigma \frac{A_{12}}{M_{s2}} - i \frac{G_{12}}{M_{s1}} \omega \right) m_{2,y} |_{x_{i3}} = 0$$

$$\left( \sigma \frac{A_1}{M_{s1}} \frac{\partial}{\partial x} - \frac{-\sigma K_1^i + A_{12}}{M_{s1}} + i \sigma \frac{G_{12}}{M_{s1}} \omega \right) m_{1,x} |_{x_{i2}} + \left( \sigma \frac{A_{12}}{M_{s2}} - i \frac{G_{12}}{M_{s1}} \omega \right) m_{2,x} |_{x_{i3}} = 0$$

### Second couple

$$\left( \frac{A_2}{M_{s2}} \frac{\partial}{\partial x} + \sigma \frac{A_{12}}{M_{s2}} - i \frac{G_{12}}{M_{s2}} \omega \right) m_{2,y} |_{x_{i3}} - \left( \frac{A_{12}}{M_{s1}} - i \sigma \frac{G_{12}}{M_{s2}} \omega \right) m_{1,y} |_{x_{i2}} = 0$$

$$\left( \frac{A_2}{M_{s2}} \frac{\partial}{\partial x} + \frac{-K_2^i + \sigma A_{12}}{M_{s2}} - i \frac{G_{12}}{M_{s2}} \omega \right) m_{2,x} |_{x_{i3}} - \left( \frac{A_{12}}{M_{s1}} - i \sigma \frac{G_{12}}{M_{s2}} \omega \right) m_{1,x} |_{x_{i2}} = 0$$

where  $A_{12}$  is static (RKKY) interlayer coupling, and

$$G_{12} = \hbar g_r^{(i)} / (8\pi) \text{ with } g_r^{(i)} \text{ being mixing conductance of the spacer}$$



# Continuity of dipolar-exchange field and magnetization

tangential component of  $\mathbf{h}_j(\mathbf{r})$  and the normal component of  $\mathbf{h}_j(\mathbf{r}) + \mathbf{m}_j(\mathbf{r})$  must be continuous across the interfaces

## External interfaces

$$\left(\frac{\partial}{\partial x} + q\right) \psi_1 - m_{1,x} |_{x_{i1}} = 0$$

$$\left(\frac{\partial}{\partial x} - q\right) \psi_2 - m_{2,x} |_{x_{i4}} = 0$$

## Internal interfaces

$$e^{-qL_s/2} \left[ \left( q - \frac{\partial}{\partial x} \right) \psi_1 + m_{1,x} \right]_{x_{i3}} = e^{qL_s/2} \left[ \left( q - \frac{\partial}{\partial x} \right) \psi_2 + m_{2,x} \right]_{x_{i2}}$$

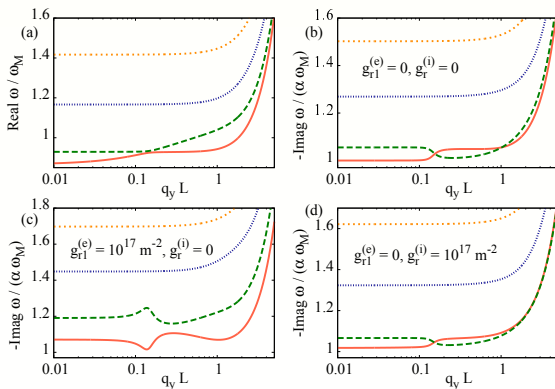
$$e^{qL_s/2} \left[ \left( q + \frac{\partial}{\partial x} \right) \psi_1 - m_{1,x} \right]_{x_{i3}} = e^{-qL_s/2} \left[ \left( q + \frac{\partial}{\partial x} \right) \psi_2 - m_{2,x} \right]_{x_{i2}}$$

# Calculation of the eigenfrequencies

- We have 12 boundary conditions with 12 unknown variables  $C_{j,1}^{(l)}$  and  $D_{j,1}^{(l)}$ .
- The system of linear equation has nontrivial solution when
$$\det \bar{\mathbf{M}} = 0$$
where  $\bar{\mathbf{M}}$  is matrix of the boundary conditions.
- we find numerically  $\omega = \omega_r + i \omega_i$  satisfying the condition.

## No interface anisotropy

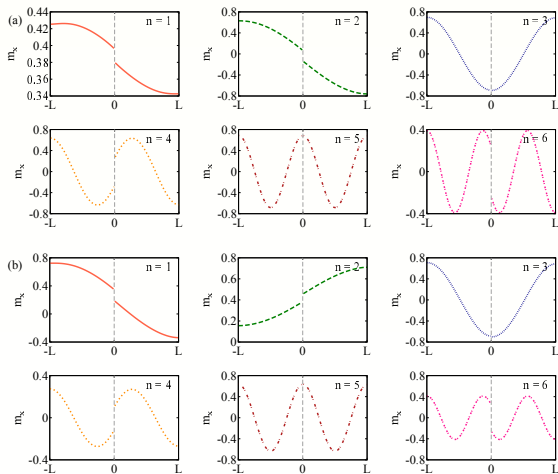
## Spin wave spectra



(a) real part of  $\omega$ , (b) no spin pumping, (c) spin pumping at the top interface, (d) spin pumping through the spacer

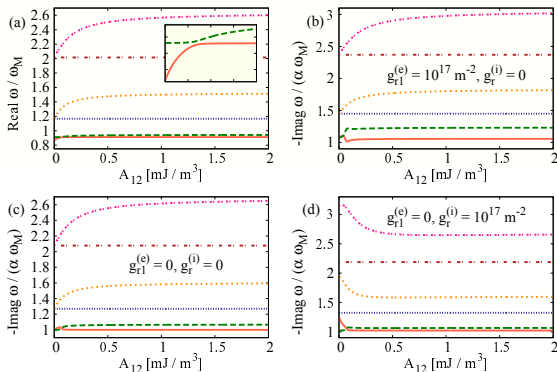
## No interface anisotropy

## Spin wave profiles

(a)  $qL = 0.1$ , (b)  $qL = 0.3$

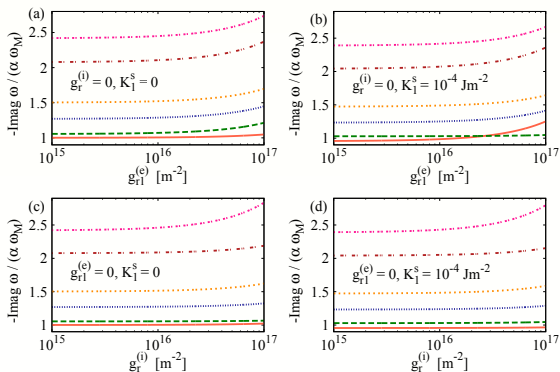
# No interface anisotropy

Dependence on interlayer coupling  $A_{12}$



(a) real part of  $\omega$ , (b) no spin pumping, (c) spin pumping at the top interface, (d) spin pumping through the spacer

## Influence of mixing conductance



(a), (c) function of spin pumping at the top interface, (b), (d) function of spin pumping through the spacer

# Summary

- Tunability of **critical current** in a spin valve with composite free layer with AF exchange coupling has been studied.
- Manipulation of **spin wave damping** by means of spin pumping in a YIG double layer has been demonstrated.

Dziękuję bardzo za uwagę